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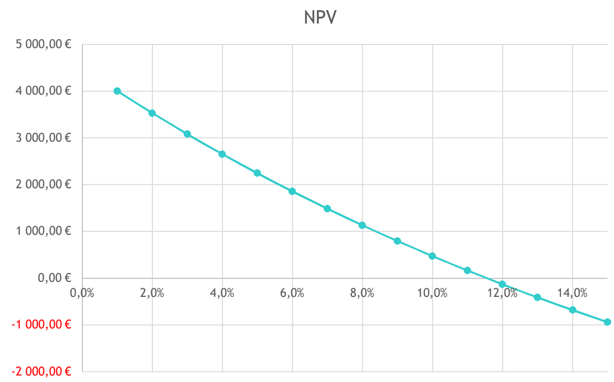
UNIVERSIDADE DE LISBOA

PITFALLS OF IRR

The NPV profile and IRR

Years	0	1	2	3	4	5
Cash flow	-10 000 €	2 000 €	2 500 €	1 000 €	4 000 €	5 000 €
Discount rate	10%					
NPV	472,27 €					
IRR	11,6%					



Pitfall 1
Not clear if you are lending or borrowing?

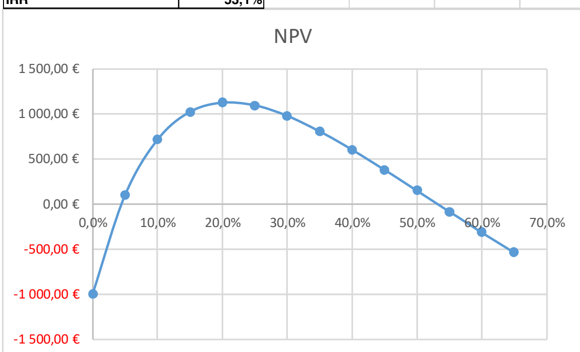
Project	0	1	2	3	IRR	NPV at 10%
A	-1 000 €	120 €	120 €	1 120 €	12,0%	45,22 €
B	1 000 €	-120 €	-120 €	-1 120 €	12,0%	-45,22 €

IRR is 12%. This is higher that cost of capital (10%).
This means that Projects A and B are equally attractive?
No!

In A we are lending money at 12%, which is good for value creation
In B we are borrowing money at 12%, which is not good for value creation

Pitfall 2
You may find projects with multiple IRR

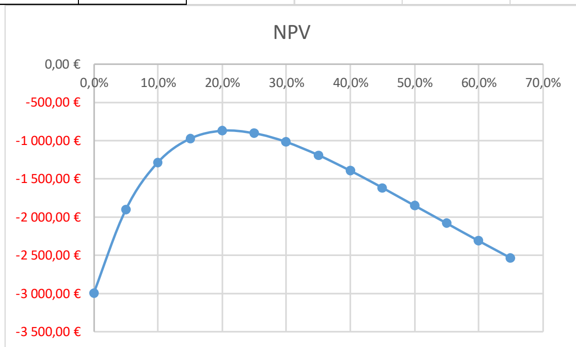
Years:	0	1	2	3	4	5
Cash flows	-7 000 €	8 000 €	2 000 €	4 000 €	12 000 €	-20 000 €
Cost of capital	10%					
NPV	**708,6 €**					
IRR	**4,5%**					
IRR	**53,1%**					



There can be as many solutions to the IRR definition as there are changes of sign in the time ordered cash flow series.

Pitfall 3
You may find projects without an IRR

Years:	0	1	2	3	4	5
Cash flows	-9 000 €	8 000 €	2 000 €	4 000 €	12 000 €	-20 000 €
Cost of capital	10%					
NPV	**-1 291,4 €**					
IRR	**#NUM!**					
IRR	**#NUM!**					



Pitfall 4 - Different timing of cash flows in mutually exclusive projects

Years	Project A	Project B	A-B
0	-1 000 €	-1 000 €	0 €
1	0 €	400 €	-400 €
2	200 €	400 €	-200 €
3	300 €	300 €	0 €
4	500 €	300 €	200 €
5	900 €	200 €	700 €
Cost of capital	10%		
NPV	**291 €**	**249 €**	42 €
IRR	**17,3%**	**20,5%**	12,5%
PI	**1,29**	**1,25**	N/D

Pitfall 5 - Different sizes of mutually exclusive projects

Years	Project A	Project B	A-B
0	-10 000	-2 000	-8 000
1	4 000	800	3 200
2	4 000	800	3 200
3	3 000	600	2 400
4	3 000	600	2 400
5	2 000	600	1 400
Cost of capital	10%		
NPV	2 487	622	1 865
IRR	20,5%	22,4%	20,0%
PI	1,25	1,31	1,23

Pitfall 6 - Unequal life spans

Years	Project A	Project B	A-B
0	-10 000 €	-10 000 €	0 €
1	3 000 €	6 400 €	-3 400 €
2	3 000 €	6 400 €	-3 400 €
3	3 000 €		3 000 €
4	3 000 €		3 000 €
5	3 000 €		3 000 €
Cost of capital	10%	10%	10%
NPV	1 372 €	1 107 €	265 €
IRR	15,2%	18,2%	12,0%
PI	1,14	1,11	N/D

The NPV shows the present value of two investments that have uneven cash flows. When comparing two different investments using the NPV method, the length of the investment (n) is not taken into consideration

In this case, is better to use the Annual Equivalent Value

Annual Equivalent Value

The equivalent annual value formula is used in capital budgeting to show the NPV of an investment as a series of equal cash flows for the length of the investment.

This is one year in financial terms= $\frac{1}{(1+k)^1}$

This is *n* years in financial terms = $A_{k;n} = \sum_{i=1}^{n} \frac{1}{(1+k)^i}$

$$A_{k;n} = \frac{1}{k}\left[1 - \frac{1}{(1+k)^n}\right]$$

So, annual equivalent value is:

$$AEV = \frac{NPV}{\sum_{i=1}^{n} \frac{1}{(1+k)^i}}$$

The calculation for projects A and B Annual Equivalent Value

$$AEV = \frac{NPV}{\sum_{i=1}^{n} \frac{1}{(1+k)^i}}$$

PROJECT A

$$A_{10\%;5} = \frac{1}{(1{,}1)^1} + \frac{1}{(1{,}1)^2} + \frac{1}{(1{,}1)^3} + \frac{1}{(1{,}1)^4} + \frac{1}{(1{,}1)^5} = 3{,}79$$

$A_{k;n} = \sum_{i=1}^{n} \frac{1}{(1+k)^i}$ $\qquad A_{10\%;5} = \frac{1}{10\%}\left[1 - \frac{1}{(1+10\%)^5}\right] = 3{,}79$

$$AEV_A = \frac{1372€}{3{,}79} = 362€/year$$

PROJECT B

$$A_{10\%;2} = \frac{1}{(1{,}1)^1} + \frac{1}{(1{,}1)^2} = 1{,}74$$

$$A_{10\%;2} = \frac{1}{10\%}\left[1 - \frac{1}{(1+10\%)^2}\right] = 1{,}74$$

$$AEV_B = \frac{1107€}{1{,}74} = 638€/year$$

$$Factor\ in\ Excel = \frac{1}{PMT(rate; n; -1€)}$$

$$AEV\ in\ Excel = NPV \times PMT(rate; n; -1€)$$

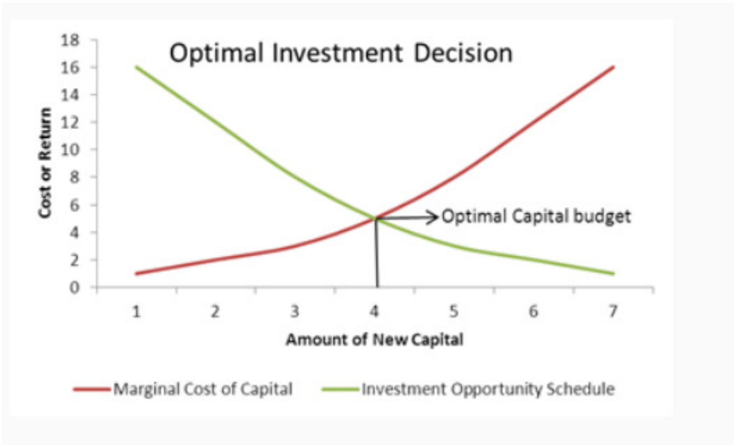
Explaining why IRR is misleading in comparison to NPV

	Cash Flow	At IRR
Reinvestment rate		22,6%
0	-65 000	
1	15 000	33 904
2	20 000	36 868
3	25 000	37 586
4	30 000	36 784
5	35 000	35 000
IRR	22,6%	
Future value		180 142
Geometric average rate of return		22,6%

IRR formula assumes that cash flow generated is reinvested at the same rate as IRR.

And this is not true, according to classical economics theory (see next slide)

Marginal cost of capital and investment schedule based on classical economics theory



The Modified IRR

We may decide the level of reinvestment rate

$$MIRR = \sqrt[n]{\frac{\sum_{i=1}^{n} FC_i \times (1+r)^{(n-i)}}{I_0}}$$

MIRR – Modified IRR
CF_i – Cash Flow at year i
r – Reinvestment rate
I_0 – Initial Investiment

	Cash Flow	At another rate
Reinvestment rate		12%
0	-65 000	
1	15 000	23 603
2	20 000	28 099
3	25 000	31 360
4	30 000	33 600
5	35 000	35 000
IRR	22,6%	
Future value		151 661
Geometric average rate of return		18,5%
Excel Formula: MIRR(range;kfinance;kreinv)		18,5%

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CAPITAL RATIONING

Profitability Index may perform better than NPV or IRR under capital rationing

Capital constraint = 100M€

Project	Investment	NPV	PI
A	40 €	20 €	1,50
B	100 €	35 €	1,35
C	50 €	24 €	1,48
D	60 €	18 €	1,30
E	50 €	10 €	1,20
Capital Constraint	100 €		

Ranking by NPV	Investment	NPV	PI
B	100 €	35 €	1,35

Is there a better solution?

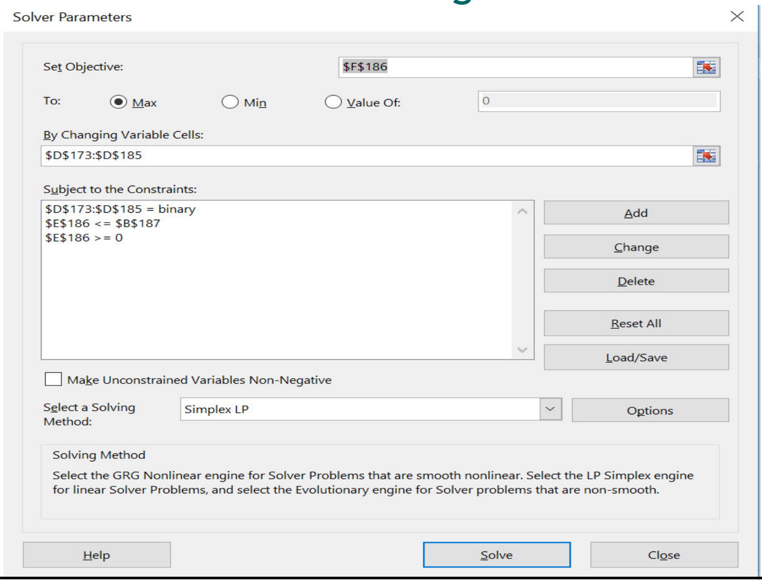
Rank by PI	Investment	NPV	PI
A	40 €	20 €	1,50
C	50 €	24 €	1,48
Liquidity	10 €		
Total NPV		44 €	1,49

We cannot choose on the basis of the NPV.
When funds are limited we need to find how to maximize the NPV.
We must pick the projects that offer the highest NPV per euro of investment outlay.

Under capital rationing linear programming maximizing NPV is a better approach

Project	Investment	NPV	Include	Selected Projects Investement	Selected Projects NPV
A	237 000	84 300	1	237 000	84 300
B	765 000	26 900	1	765 000	26 900
C	304 000	23 200	1	304 000	23 200
D	565 000	82 600	1	565 000	82 600
E	109 000	20 500	1	109 000	20 500
F	89 000	90 400	1	89 000	90 400
G	796 000	18 200	1	796 000	18 200
H	814 000	97 600	1	814 000	97 600
I	480 000	52 000	1	480 000	52 000
J	827 000	54 000	1	827 000	54 000
K	734 000	56 300	1	734 000	56 300
L	911 000	88 300	1	911 000	88 300
M	978 000	69 400	1	978 000	69 400
Total	**7 609 000**	**763 700**	**13**	**7 609 000**	**763 700**
Constraint	**3 000 000**				

Solver Parameters using Excel



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The solution using Solver of Excel

				Selected Projects	
Project	Investment	NPV	Include	Investement	NPV
A	237 000	84 300	1	237 000	84 300
B	765 000	26 900	0	0	0
C	304 000	23 200	1	304 000	23 200
D	565 000	82 600	1	565 000	82 600
E	109 000	20 500	0	0	0
F	89 000	90 400	1	89 000	90 400
G	796 000	18 200	0	0	0
H	814 000	97 600	1	814 000	97 600
I	480 000	52 000	0	0	0
J	827 000	54 000	0	0	0
K	734 000	56 300	0	0	0
L	911 000	88 300	1	911 000	88 300
M	978 000	69 400	0	0	0
Total	**7 609 000**	**763 700**	6	**2 920 000**	**466 400**
Constraint	**3 000 000**				

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But life can be more complex than that

Multi-period analysis

Projects	Cash flows			NPV	PI
	0	1	2		
A	-10,0 €	30,0 €	5,0 €	21,4 €	3,14
B	-5,0 €	5,0 €	20,0 €	16,1 €	4,21
C	-5,0 €	5,0 €	15,0 €	11,9 €	3,39
D		-40,0 €	60,0 €	13,2 €	1,33

According to PI you must should A and B = 16,1€+11,9€= 28,0€

But if you choose A in year 0, you may choose D in year 1
A+B=21,4€+13,2€=34,6€

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FINAL COMMENTS

Basic rules for financial decision

- QUANTIFY the relevant cash flow for each year;
- Identify the level of RISK of cash flows and decide the appropriate discount rate considering the level of risk;
- Discount the cash flows of each project with the relevant discount rate;
- Compare the NPV of each project at the same time value of money.

Investment decision is not a black blox?

- Net operating cash flow (cash flow to the firm) or net cash flow (cash flow to the equity)?
- Incremental cash flows
  - Do not confuse average with incremental cash flows
  - Include all incidental effects
  - Do not forget working capital requirements
  - Include opportunity costs
  - Forget the sunk costs
  - Beware of allocated overhead costs
- Treat inflation consistently
- Separate investment from financing decisions
- Depreciation is a non-cash expense. It is important only because it is tax deductible

Treat inflation consistently

Cash flows in real terms

	0	1	2	3
Cash flows (real terms)	-1 000 €	300 €	500 €	400 €
Cost of capital (real terms)	6%			
NPV	**63,86 €**			
IRR	**9,3%**			

Cash flows in nominal terms

	0	1	2	3
Inflation rate	2,50%			
Cash flows (nominal terms)	-1 000 €	308 €	525 €	431 €
Cost of capital (nominal terms)	9%			
NPV	**63,86 €**			
IRR	**12,0%**			
IRR (real terms)	**9,3%**			

Fisher Formula: $r_n = (1 + r_r) \times (1 + r_i)$

r_n – nominal rate
r_r – rate in real terms
r_i = inflation rate